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Critical-point behaviour of axially deformed nuclei in the octupole degree of freedom

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Abstract. The evolution of the octupole excitation is investigated along the Th isotope chain. The isotope ^{226}Th results to be close to the critical point (square-well potential in the octupole amplitude β_3).

PACS. 21.60.Ev Collective models

Evidence of phase transition between spherical and axially deformed shape —and examples of the critical-point symmetry $X(5)$ introduced by Iachello [1]— have been recently found [2–4] in two regions of nuclei: around ^{104}Mo ($Z=42$, $N=62$) and around ^{152}Sm ($Z=62$, $N=90$). We now observe that a similar transition takes place also in the region of $Z = 90$ (Th isotopes), as shown in fig. 1a. Here, however, octupole excitations of very low energy are also apparent (fig. 1b). Therefore, in investigating the evolution of nuclear shapes in this region, we cannot avoid to take into account at the same time the quadrupole and the octupole modes. This work is in progress [5]. Here, we limit the discussion to a subset of data, concerning the evolution of the octupole mode in nuclei which already possess a stable quadrupole deformation.

Also in this limited field, one can meet a variety of situations: at the larger values of N, the octupole vibration is almost decoupled from the quadrupole modes, and gives rise to bands with $K^\pi = 0^-, 1^-, 2^-, 3^-$ (presumably coupled together by the Coriolis forces), at excitation energies not far from those of the β and γ quadrupole bands. When N decreases, the $K^\pi = 0^-$ band (and only this one) moves down, and tends to merge with the g.s. band into an alternate-parity band, similar to those of asymmetric diatomic molecules. At this limit, the octupole and quadrupole modes appear to be coupled together, being constrained to maintain a common symmetry axis, while the Coriolis coupling of the axial octupole mode with non-axial ones should be weaker, due to the large energy denominator. We can relate this behavior with an evolution of the potential for the amplitude β_3 of axial octupole deformation, from a shape with two symmetric minima to a single minimum at $\beta_3 = 0$. A “flat” shape —as for the $X(5)$ potential of the quadrupole— can be expected at a critical point in between. Actually, the HFB-

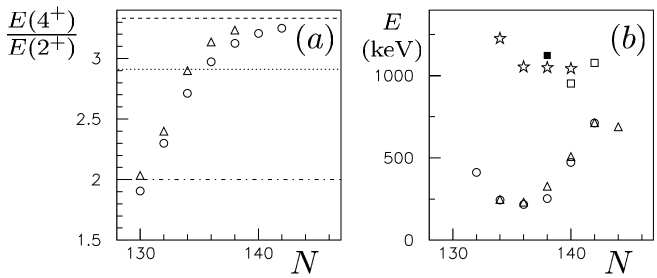

Fig. 1. a): Ratios $E(4^+)/E(2^+)$ for isotopes of Th (triangles) and Ra (circles); dotted line: critical $X(5)$ value. b): $E(1^-)$ for Th (triangles) and Ra (circles), compared with $E(1^-_2)$ (open symbols) or $E(2^-)$ (full symbol), for Th (squares) or Ra (stars). Data are from the NNDC data base [6].

Cranking model calculations by Nazarewicz and coworkers [7] show that all these situations can be found in different Th isotopes.

There are several theoretical frames to treat at the same time the quadrupole and octupole degrees of freedom. An *algebraic scheme* is given by the *spdf* interacting-boson model [8–10]. In the *geometrical model* (more suitable for the present purposes) a completely consistent treatment in the full model space is only provided by the theory of Donner and Greiner [11–13]. In alternative, a number of simplified models, usually limited to axial octupole vibrations, have been proposed [13–17].

In the model we are going to use here, we assume:

i) Permanent quadrupole deformation $\bar\beta_2$;
ii) Amplitude of β_2 vibrations (around $\bar\beta_2$) and of γ vibrations (around zero) negligible in comparison to $\bar\beta_2$;
iii) Axial octupole vibrations, in a proper potential well;
iv) Amplitude of non-axial octupole vibrations ($K \neq 0$) negligible in comparison to $\bar\beta_2$;
v) Rotation-vibration wave function $\Psi = \psi(\beta_3)\ \mathcal{D}^{J*}_{M,0}$, with $\psi(-\beta_3) = (-)^J \psi(\beta_3)$.

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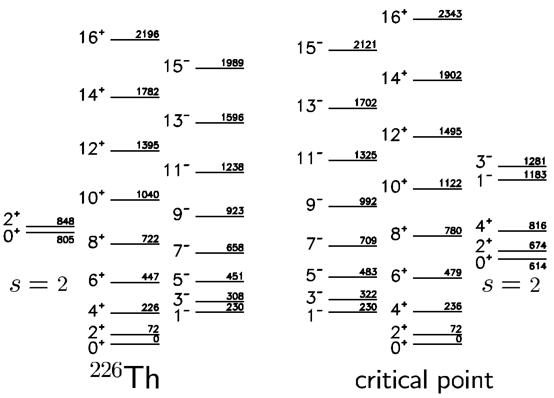

Fig. 2. Level scheme of ^{226}Th *vs.* model predictions ($b = 1.73$).

With the new variable $x = \sqrt{2B_3/B_2}(\beta_3/\bar{\beta}_2)$ —where B_λ is the inertia parameter [12]— and $\epsilon = (1/\hbar^2)B_2\,\bar{\beta}_2^2\,E$, we obtain the Schrödinger equation

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{2x}{1+x^2}\,\frac{\mathrm{d}\psi}{\mathrm{d}x} + \left[\epsilon - \frac{J(J+1)}{6(1+x^2)} - V(x)\right]\psi = 0\,,$$

which, for $V(x) = 0$, is formally equivalent to that of *Oblate spheroidal wave functions* [18] with $m = 0$, $c^2 = \epsilon$ and $\lambda = J(J+1)/6 - \epsilon$. The critical point corresponds to $V(x) = 0$ for $|x| < b$ and $= +\infty$ for $|x| > b$, *i.e.* $c = 0$ and boundary conditions $\psi(\pm b) = 0$. Obviously, in this case the results depend on the parameter b, that can be adjusted to reproduce, *e.g.*, the position of the first 1^- level.

At this point, a word of caution is obviously in order. Quantizing the kinetic-energy operator in non-Cartesian coordinates is a difficult (and insecure) task. Here, we are discussing a *model* that can find its justification in the (possible) agreement with experimental data. However:

i) In the low-amplitude limit, our expression coincides with the corresponding limit of Donner and Greiner [12];
ii) the only relevant assumption concerns the variable moment of inertia $\mathcal{J}_1 = 3B_2\bar{\beta}_2^2 + 6B_3\beta_3^2$, while other details of the model have practically no effect in the region of interest.

As we have seen, for the validity of the model one needs a permanent quadrupole deformation —*i.e.* $E(4^+)/E(2^+)$ not far from 3.3)— *and* axial ($K{=}0$) octupole excitation well below those with $K{\neq}0$ —*i.e.* $E(1_2^-)$ (and $E(2^-)$) $\gg E(1^-)$. A glance to the two sections of fig. 1 shows that these two conditions are fulfilled at the same time only in a very limited interval of values of N, so that our analysis must be limited to the two isotopes ^{226,228}Th.

Both the positive- and the negative-parity parts of the g.s. band of ^{226}Th (fig. 2) result to be in rather good agreement with the model predictions at the critical point, with $b = 1.73$ (fit on the lowest 1^- level). As shown in fig. 3a, the overall agreement is apparently improved with $b = 1.87$ (fit on the 20^+ level), at the expense of a slight deviation between the theoretical and the experimental value for the 1^- level. For the 0_2^+ and 2_2^+ levels of the $s = 2$ band the agreement is not so good, but not substantially worse than for "good" $X(5)$ nuclei as ^{104}Mo and ^{150}Nd. Branching ratios for the $E1$ transitions (table 1) provide a further test of the model.

At lower values of N, the assumption of permanent quadrupole deformation appears to fail already for ^{224}Th ($N = 134$) and, quite surprisingly, the positive part of the g.s. band of ^{224}Th —and also of ^{224}Ra— follows rather well the pattern typical of $X(5)$ symmetry [5]. At the opposite side, the level scheme of ^{228}Th (fig. 3b) is, at least up to $J = 16$, in rough agreement with the model predictions for a harmonic potential, thus confirming the validity of the basic assumptions of our model.

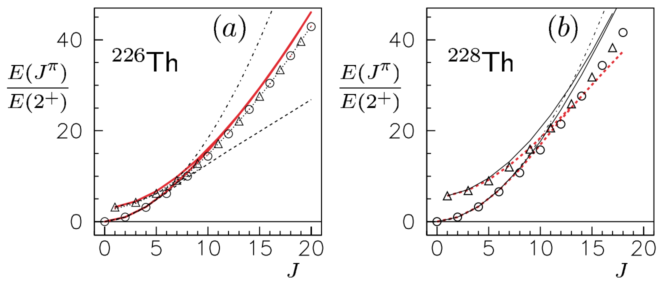

Fig. 3. Experimental data for ^{226}Th and ^{228}Th (circles: positive-parity levels; triangles: negative parity) compared with model predictions, fitted on the 1^- level. Full line: critical potential; dashed line: harmonic potential; dot-dashed line: rigid rotor. The dotted line in a) corresponds to the critical-potential fit on the 20^+ level.

Table 1. Ratios of reduced strengths for $E1$ transitions coming from the same level of ^{226}Th. Theoretical values are calculated assuming [11] $\mathcal{M}_\mu(E1) \propto \beta_2\beta_3 Y_{1,\mu}$.

J_i^π	Trans. 1	$\Rightarrow J_{f1}^\pi$	Trans. 2	$\Rightarrow J_{f2}^\pi$	$B_{i\to f1}(E1)/B_{i\to f2}(E1)$ Theor.	Experimental
1^-	$E1$	0^+	$E1$	2^+	0.47	0.54 ± 0.05
3^-	$E1$	2^+	$E1$	4^+	0.65	0.99 ± 0.25
2_2^+	$E1$	1^-	$E1$	3^-	0.63	0.60 ± 0.18

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